

**Quadratics**

**Calculators may NOT be used to answer these questions unless a symbol is shown next to the question.**

**1.** (i)  Show that *x*2 – 8*x* + 17 > 0 for all real values of *x*

(3)

(ii)  "If I add 3 to a number and square the sum, the result is greater than the square

of the original number."

 State, giving a reason, if the above statement is always true, sometimes true or never true.

(2)

(Total for question 1 is 5 marks)

**2.** f (*x*) = *x*2 – 10*x* + 23

 (*a*)Express f (*x*) in the form (*x* + *a* )2 + *b*, where *a* and *b* are constants to be found.

(2)

 (*b*)Hence, or otherwise, find the exact solutions to the equation

*x*2 – 10*x* + 23 = 0

(2)

(Total for Question 2 is 4 marks)

**3.** Factorise completely *x* − 4*x*3.

(3)

(Total for Question 3 is 3 marks)



 **4.** The equation *kx*2 + 4*kx* + 3 = 0, where *k* is a constant, has no real roots.

 Prove that 0 ≤ *k* < 

 (4)

(Total for question 4 is 4 marks)

**5.** The curve *C* has equation

*y* = + 1 *x* ∈ ℝ, *x* ≠ 0

where *k* is a constant.

The line *l* has equation *y* = –2*x* + 5

 (*a*)  Show that the *x* coordinate of any point of intersection of *l* with *C* is given by a solution of the equation

2*x*2 – 4*x* + *k*2 = 0

(2)

 (*b*)  Hence find the exact values of *k* for which *l* is a tangent to *C*.

(3)

(Total for question 5 is 5 marks)

**6.** Find, using algebra, all real solutions to the equation *b*4 + 7*b*2 – 18 = 0

(4)

(Total for question 6 is 4 marks)

**7.** A company started mining tin in Riverdale on 1st January 2019.

A model to find the total mass of tin that will be mined by the company in Riverdale is given by the equation

*T* = 1200 – 3(*n* – 20)2

where *T* tonnes is the total mass of tin mined in the *n* years after the start of mining.

Using this model,

 (*a*)  calculate the mass of tin that will be mined up to 1st January 2020,

(1)

 (*b*)  deduce the maximum total mass of tin that could be mined,

(1)

 (*c*)  calculate the mass of tin that will be mined in 2023.

(2)

 (*d*)  State, giving reasons, the limitation on the values of *n*.

(2)

(Total for question 7 is 6 marks)

**8.**



A company makes a particular type of children's toy.

The annual profit made by the company is modelled by the equation

*P* = 100 – 6.25(*x* – 9)2

where *P* is the profit measured in thousands of pounds and *x* is the selling price of the toy in pounds.

A sketch of *P* against *x* is shown in Figure 1.

Using the model,

 (*a*)  explain why £15 is not a sensible selling price for the toy.

(2)

Give that the company made an annual profit of more than £80 000

 (*b*)  find, according to the model, the least possible selling price for the toy.

(3)

The company wishes to maximise its annual profit.

State, according to the model,

 (*c*)  (i)  the maximum possible annual profit,

 (ii)  the selling price of the toy that maximises the annual profit.

(2)

 (Total for question 8 is 7 marks)